(DPHY 01)

M.Sc.(Previous) DEGREE EXAMINATION, DECEMBER 2006

PHYSICS :: FIRST YEAR

Paper I - MATHEMATICAL PHYSICS

Time : Three hours　　　　　　　　　　　　　　　　　　　Maximum : 100 marks

Answer any FIVE questions

Each question carry equal marks

1. (a) Obtain the solution for Hamite's differential equation. (12)

   (b) Starting from the generating function of Laguerre polynomial. Prove

$$(n+1)L_{n+1}(x)-(2n+1-x)L_n(x)-nL_{n-1}(x)=0 \qquad (8)$$

2. (a) Standing from the Legerdre's differential equation, prove the ortho normal property. (12)

   (b) Evaluate $J_{1/2}(x)$ and $J_{-1/2}(x)$, hence show that $J_{5/2}(x)=\sqrt{\frac{2}{\pi x}}\left\{\frac{\sin x}{x}-\cos x\right\}$. (8)

3. (a) State and prove Taylor's theorem. (10)

   (b) Find Taylor's series for the function: (2 x 5 = 10)

      (i) z^3-3z^2+4z-2 about $z=2$

      (ii) $\frac{1}{(1+z)}$ about $z=1$.

4. (a) Obtain Cauchy-Riemann equations in polar co-ordinates. (10)

   (b) Prove that $u=e^{-\lambda}(x\sin y-y\cos y)$ is hormonic and find v such that $f(z)=u+iv$ is analytic. (10)

5. (a) Explain the process of contraction of a tensor with one example. (8)

   (b) State and prove the rules governing the tensor analysis. (12)

6. (a) Find the components of vector in polar co-ordinates when their components in Cartesian co-ordinates as $(\dot{x},\dot{y})$ and $(\ddot{x},\ddot{y})$ respectively. (12)

   (b) Prove that $g_{\alpha\beta,\gamma}=0$. (8)

7. (a) State and prove the first and second shiffing property of Laplace transforms and hence find $L\left\{e^{-3t}\sin 2t\right\}$ and $L\left\{G(t)\right\}$, where

$$G(t) = \cos\left(t - \frac{2\pi}{3}\right), \quad t > \frac{2\pi}{3}$$

$$= 0, \qquad t < \frac{2\pi}{3} \qquad (10)$$

(b) Find

(i) $L^{-1}\left\{\frac{6}{2s-3} - \frac{3+4s}{9s^2-16} + \frac{8-6s}{16s^2+9}\right\}$

(ii) $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$ (2 x 5 = 10)

8. (a) Starting from the general Fourier series of a periodic function $f(x)$ in the interval $(-l, l)$, obtain the Fourier integral. (10)

(b) (i) Find Fourier integral of $f(x) = e^{-kx}$ when $x > 0$, and $f(x) = f(-x)$ for $k > 0$. (4)

(ii) Find Fourier transform of (6)

$$f(x) = x, |x| < a$$

$$= 0, |x| > 0$$

9. (a) Answer any TWO of the following: (2 x 10 = 20)

(a) State and prove Cauchy's integral theorem.

(b) Show that

$P_n'(x) = (2n-1)P_{n-1}(x) + (2n-5)P_{n-3}(x) + ... + 3P_1$ or P_0 according as n is even or odd.

(c) Evaluate $L^{-1}\left\{\frac{6s^2 + 22s + 18}{s^3 + 6s^2 + 11s + 6}\right\}$ using partial fractions method.

(d) State and prove quotient law for tensors.